A Counterexample to Thiagarajan's Conjecture on Regular Event Structures

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We provide a counterexample to a conjecture by Thiagarajan [8, 9] that regular event structures correspond exactly to event structures obtained as unfoldings of finite 1-safe Petri nets. Event structures, trace automata, and Petri nets are fundamental models in concurrency theory. There exist nice interpretations of these structures as combinatorial and geometric objects and both conjectures can be reformulated in this framework. Namely, from a graph theoretical point of view, the domains of prime event structures correspond exactly to median graphs; from a geometric point of view, these domains are in bijection with CAT(0) cube complexes.

A necessary condition for the conjecture to be true is that domains of regular event structures admit a regular nice labeling (which corresponds to a special coloring of the hyperplanes of the associated CAT(0) cube complex). To disprove these conjectures, we describe a regular event domain that does not admit a regular nice labeling. Our counterexample is derived from an example by Wise [10, 11] of a nonpositively curved square complex $\mathbf{X}$ with six squares, whose edges are colored in five colors, and whose universal cover $\widetilde{\mathbf{X}}$ is a CAT(0) square complex containing a particular plane with an aperiodic tiling. We prove that other counterexamples to Thiagarajan's conjecture arise from aperiodic 4-way deterministic tile sets of Kari and Papasoglu [6] and Lukkarila [7].

On the positive side, we show that event structures obtained as unfoldings of finite 1-safe Petri nets correspond to the finite special cube complexes. This subclass of nonpositively curved cube complexes was introduced by Haglund and Wise [4, 5] in geometric group theory and is characterized by simple combinatorial properties satisfied by the hyperplanes. Using the breakthrough results by Agol [1] based on special cube complexes, we prove that Thiagarajan's conjecture is true for regular event structures whose domains occur as principal filters of hyperbolic CAT(0) cube complexes which are universal covers of finite nonpositively curved cube complexes.

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