

Certain Query Answering on Compressed String Patterns: from Streams to Hyperstreams

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- Hyperstreams as a generalization of streams [Maneth 2015], [Labath 2013]
- Certain Query Answering (CQA) [David 2010]
- Nondeterministic Finite Automata (NFAs) as regular path queries [Angles 2016]

From Streams to Hyperstreams

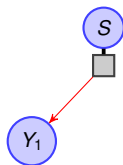
Empty stream

$$S = Y_1$$

From Streams to Hyperstreams

Empty stream

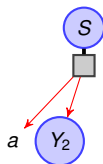
$$S = Y_1$$



From Streams to Hyperstreams

$$Y_1 \rightarrow aY_2$$

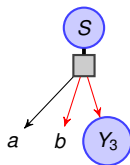
$$S = aY_2$$



From Streams to Hyperstreams

$Y_1 \rightarrow aY_2, Y_2 \rightarrow bY_3,$

$$S = abY_3$$

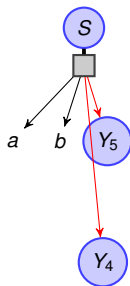


From Streams to Hyperstreams

$Y_1 \rightarrow aY_2, Y_2 \rightarrow bY_3,$

$Y_3 \rightarrow Y_4 Y_5$

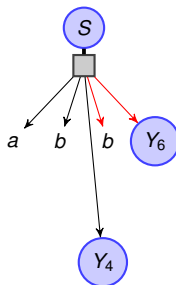
$S = abY_4 Y_5$



From Streams to Hyperstreams

$Y_1 \rightarrow aY_2, Y_2 \rightarrow bY_3,$
 $Y_3 \rightarrow Y_4Y_5, Y_5 \rightarrow bY_6,$

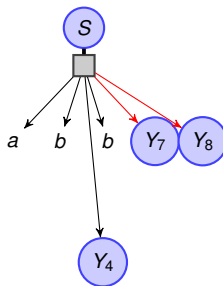
$$S = abY_4bY_6$$



From Streams to Hyperstreams

$Y_1 \rightarrow aY_2, Y_2 \rightarrow bY_3,$
 $Y_3 \rightarrow Y_4Y_5, Y_5 \rightarrow bY_6,$
 $Y_6 \rightarrow Y_7Y_8$

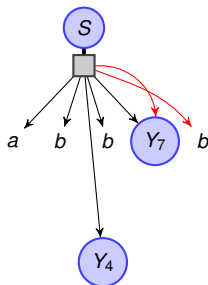
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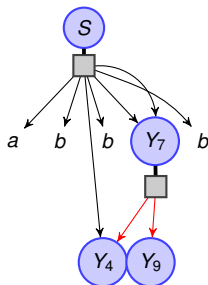
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From Streams to Hyperstreams

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 $Y_6 \rightarrow Y_7 Y_8, Y_8 \rightarrow bY_7,$
 $Y_7 \rightarrow Y_4 Y_9$

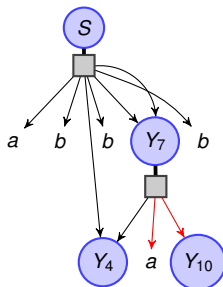
$S = abY_4bY_4Y_9bY_4Y_9$



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$Y_1 \rightarrow aY_2, Y_2 \rightarrow bY_3,$
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 $Y_7 \rightarrow Y_4 Y_9$
 $, Y_9 \rightarrow aY_{10},$

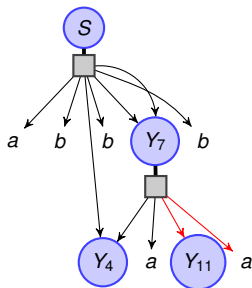
$S = abY_4bY_4aY_{10}bY_4aY_{10}$



From Streams to Hyperstreams

$Y_1 \rightarrow aY_2, Y_2 \rightarrow bY_3,$
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 $Y_7 \rightarrow Y_4 Y_9$
 $, Y_9 \rightarrow aY_{10},$
 $Y_{10} \rightarrow Y_{11} a$

$S = abY_4 bY_4 a Y_{11} abY_4 a Y_{11} a$

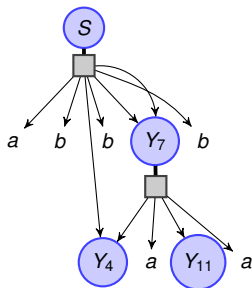


From Streams to Hyperstreams

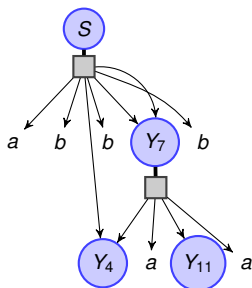
$Y_1 \rightarrow aY_2, Y_2 \rightarrow bY_3,$
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Hyperstreams are compressed
string patterns.

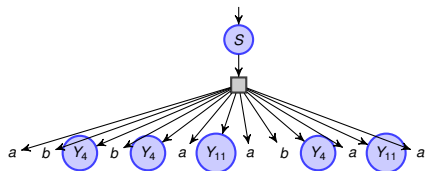
$$S = abY_4bY_4aY_{11}abY_4aY_{11}a$$



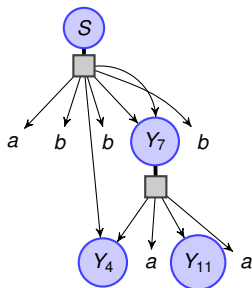
From Streams to Hyperstreams

$$\begin{aligned} Y_1 &\rightarrow aY_2, Y_2 \rightarrow bY_3, \\ Y_3 &\rightarrow Y_4Y_5, Y_5 \rightarrow bY_6, \\ Y_6 &\rightarrow Y_7Y_8, Y_8 \rightarrow bY_7, \\ Y_7 &\rightarrow Y_4Y_9 \\ , Y_9 &\rightarrow aY_{10}, \\ Y_{10} &\rightarrow Y_{11}a \end{aligned}$$
$$S = abY_4bY_4aY_{11}abY_4aY_{11}a$$
$$G = (N, \Sigma, R, S)$$
$$N = \{S, Y_4, Y_7, Y_{11}\}$$
$$R = \left\{ \begin{array}{l} S \rightarrow abY_4bY_7bY_7, \\ Y_7 \rightarrow Y_4aY_{11}a \end{array} \right\}$$


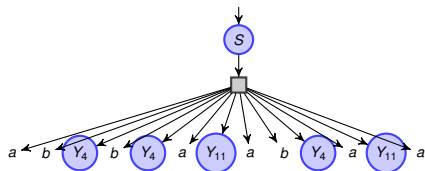
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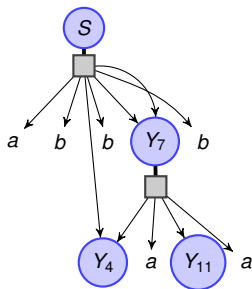


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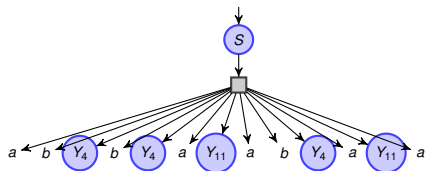
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Represents $pat(G)$



From Streams to Hyperstreams



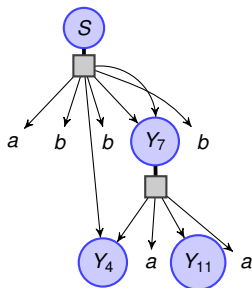
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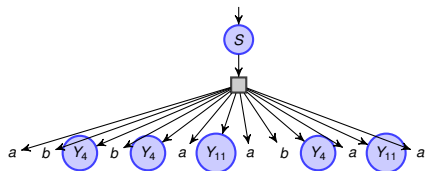
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Represents $pat(G)$

$$Inst(G) = Inst(pat(G))$$



From Streams to Hyperstreams



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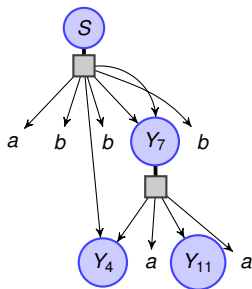
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Represents $pat(G)$

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Not linear



- 1 Certain Query Answering on hyperstreams of words
- 2 Decidability and Complexity of Certain Query Answering on Compressed String Patterns
- 3 Future Work

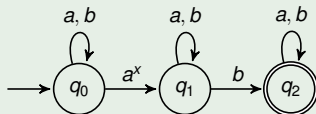
- 1 Certain Query Answering on hyperstreams of words
 - Queries on Hyperstreams
 - Regular Compressed Pattern Matching and Inclusion
 - Results
- 2 Decidability and Complexity of Certain Query Answering on Compressed String Patterns
- 3 Future Work

Example (Monadic query)

$Q_1(w)$ = Set of positions of w labeled by a and followed by b .

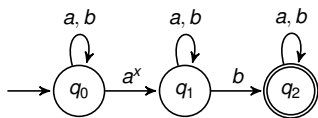
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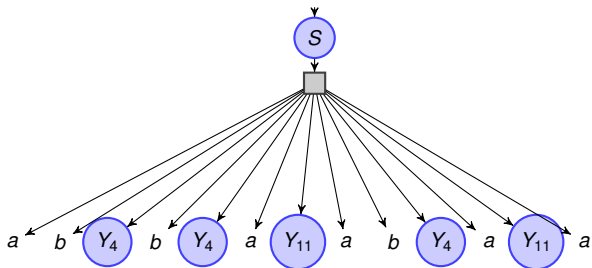
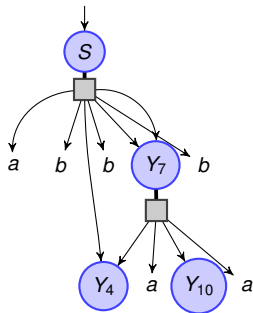
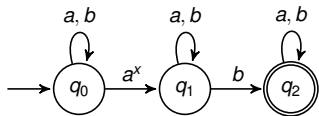


Certain (non) Answers

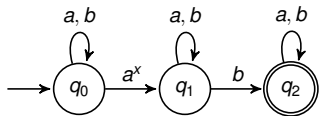
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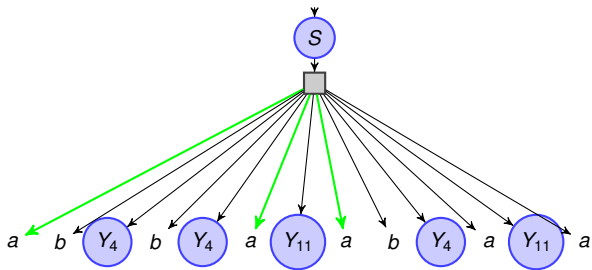
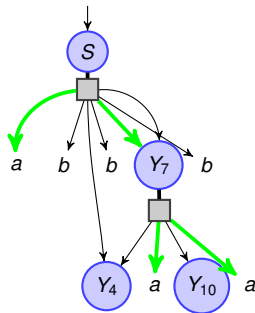
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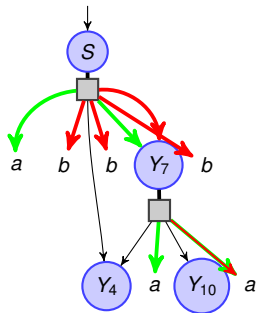
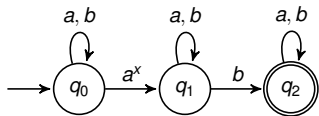
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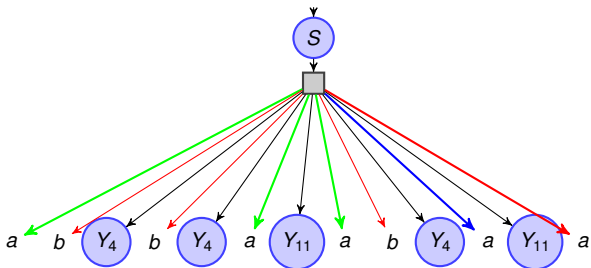


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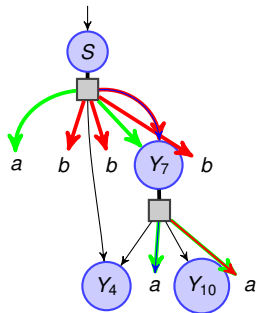
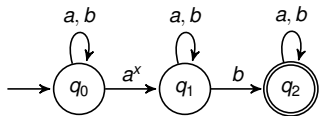


Certain answers

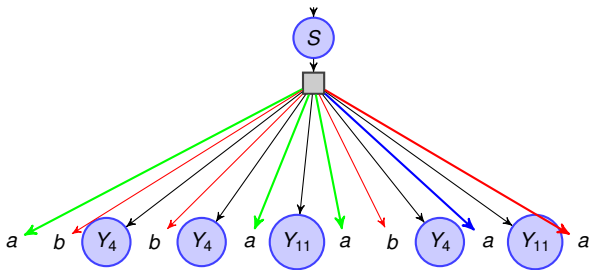
Certain non answers



Certain (non) Answers



Certain answers
Certain non answers
Neither certain nor
certain non answers



Certain query answering problem

Input: query automaton A , hyperstream G , tuple of positions π of symbols of Σ on $pat(G)$.

Output: π certain answer for $L(A)$ on G ?

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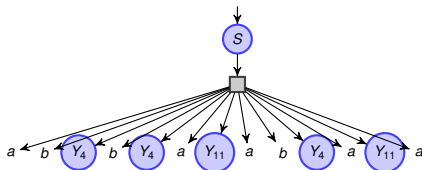
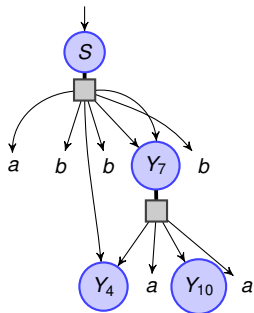
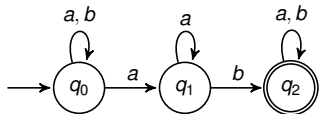
Certain query non answering problem

Input: query automaton A , hyperstream G , tuple of positions π of symbols of Σ on $pat(G)$.

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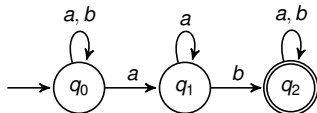
Equivalences in the boolean case

Boolean query automaton A , hyperstream G

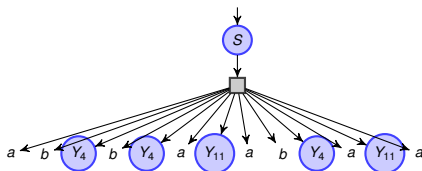
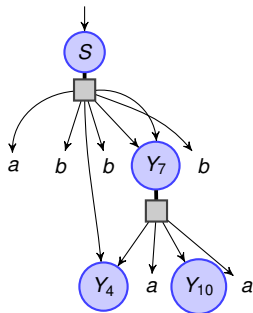


Equivalences in the boolean case

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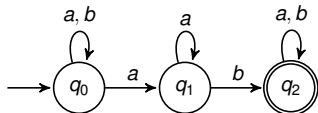


() certain answer iff
 $Inst(G) = Inst(pat(G)) \subseteq L(A)$



Equivalences in the boolean case

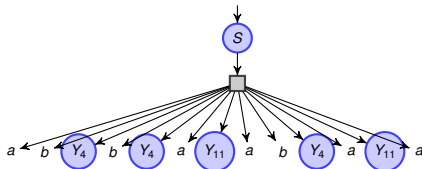
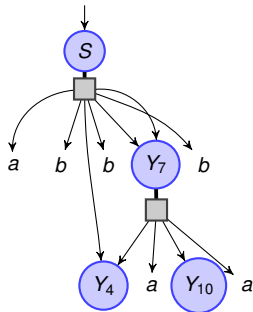
Boolean query automaton A , hyperstream G



$$CQA_{bool} = REGINCL$$

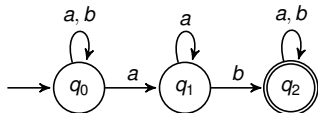
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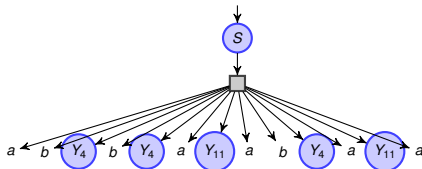
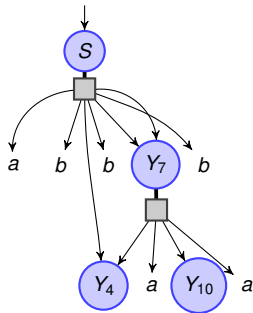
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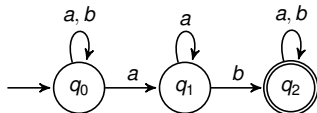
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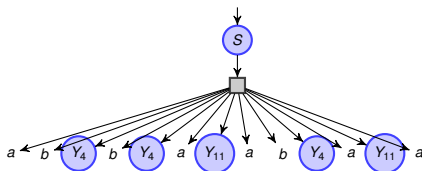
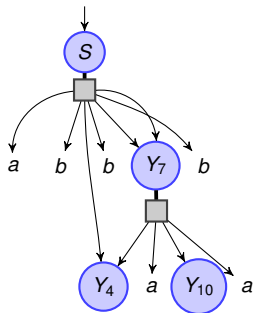
() certain answer iff

$$Inst(G) = Inst(pat(G)) \subseteq L(A)$$

$CQ_{\neg A_{bool}} = coREGMATCH$

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Known results (Complexity of CQA on streams [Gauwin 2009])

	DFAs	NFAs
Answers	P _{TIME}	PSPACE-c
Non-answers	P _{TIME}	P _{TIME}

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	DFAs	NFAs
Answers	PSPACE-C	PSPACE-C
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Contribution (Complexity of CQA on **Hyperstreams**)

	DFAs	NFAs
Answers (<i>REGINCL</i>)	PSPACE-C	PSPACE-C
Non-answers (<i>coREGMATCH</i>)	PSPACE-C	PSPACE-C

Known results (Complexity of CQA on streams [Gauwin 2009])

	DFAs	NFAs
Answers	PTIME	PSPACE-C
Non-answers	PTIME	PTIME

Contribution (Complexity of CQA on **linear hyperstreams**)

	DFAs	NFAs
Answers (<i>REGINCL</i>)	PTIME	PSPACE-C
Non-answers (<i>coREGMATCH</i>)	PTIME	PTIME

- 1 Certain Query Answering on hyperstreams of words
- 2 Decidability and Complexity of Certain Query Answering on Compressed String Patterns
 - General case
 - An efficient fragment
- 3 Future Work

Lemma

REGINCL(Linear_String_patterns, NFAs) is PSPACE-hard.

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Proof.

String pattern Y and NFA A

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String pattern Y and NFA A

$Inst(Y) \subseteq L(A)$ iff $L(A) = \Sigma^*$.

Hardness of regular compressed pattern inclusion

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REGINCL(Linear_String_patterns, NFAs) is PSPACE-hard.

Proof.

String pattern Y and NFA A

$Inst(Y) \subseteq L(A)$ iff $L(A) = \Sigma^*$.

PSPACE-complete [Kozen 1977]. □

Contribution (Complexity of CQA on **Hyperstreams**)

	DFAs	NFAs
Answers (<i>REGINCL</i>)	-	-
Non-answers (<i>coREGMATCH</i>)	-	-

Contribution (Complexity of CQA on **linear hyperstreams**)

	DFAs	NFAs
Answers (<i>REGINCL</i>)	-	PSPACE-hard
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Contribution (Complexity of CQA on **linear hyperstreams**)

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Answers (<i>REGINCL</i>)	-	PSPACE-hard
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Theorem ([Angluin 1980])

The simpler problem of string pattern matching is NP-complete.

Complexity of regular compressed pattern matching

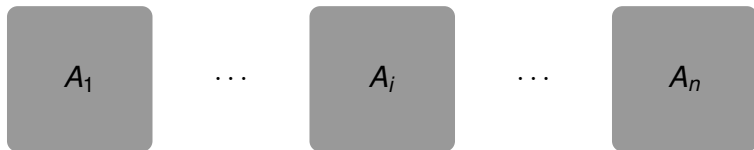
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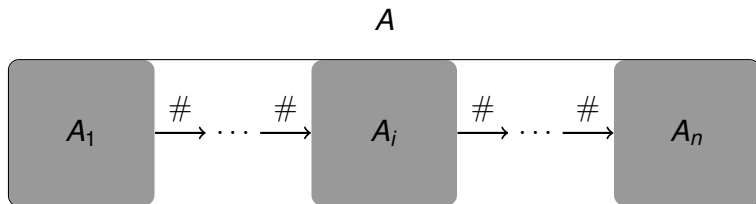
Theorem

REGINCL(String_patterns, DFAs) is PSPACE-complete.

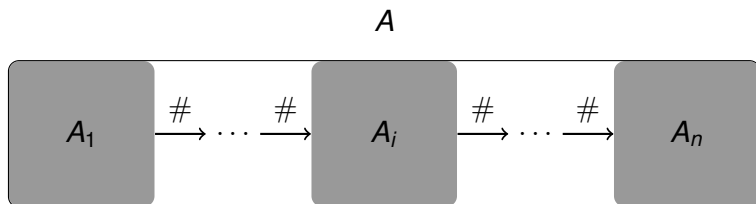
Hardness of regular compressed pattern matching



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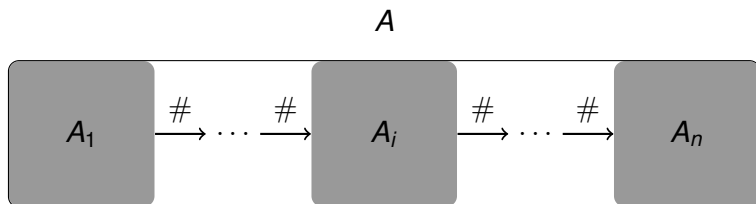


Hardness of regular compressed pattern matching



$$L(A) = \{u_1\#\dots\#u_n \mid u_1 \in L(A_1), \dots, u_n \in L(A_n)\}$$

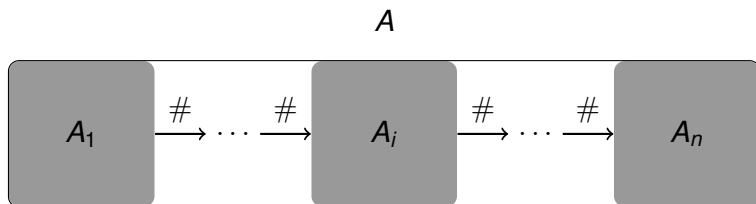
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$$\text{String pattern } p = \underbrace{Y \# \dots \# Y}_n$$

Hardness of regular compressed pattern matching

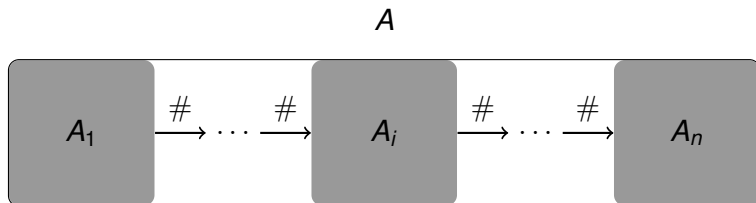


$$L(A) = \{u_1 \# \dots \# u_n \mid u_1 \in L(A_1), \dots, u_n \in L(A_n)\}$$

$$\text{String pattern } p = \underbrace{Y \# \dots \# Y}_n$$

$$\text{Inst}(p) \cap L(A) \neq \emptyset \text{ iff } L(A_1) \cap \dots \cap L(A_n) \neq \emptyset.$$

Hardness of regular compressed pattern matching



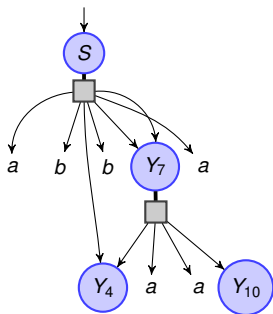
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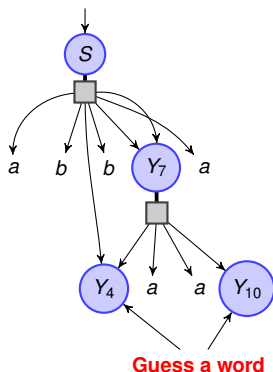
$$\text{Inst}(p) \cap L(A) \neq \emptyset \text{ iff } L(A_1) \cap \dots \cap L(A_n) \neq \emptyset.$$

PSPACE-complete [Kozen 1977].

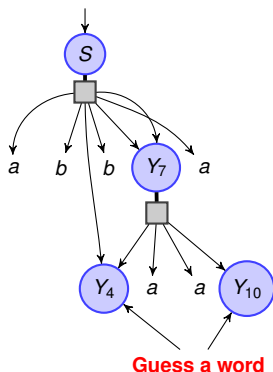
How to decide regular compressed pattern matching ?



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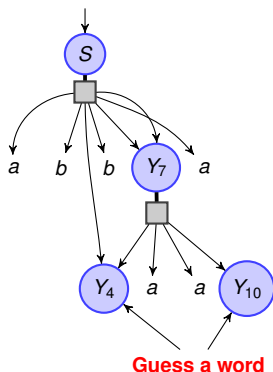


How to decide regular compressed pattern matching ?



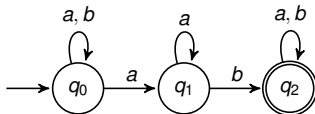
Σ^* is not finite...

How to decide regular compressed pattern matching ?

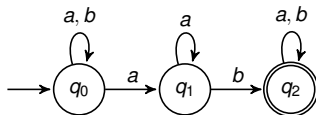


Σ^* is not finite... but the transition monoid is !

Transition monoid



Transition monoid



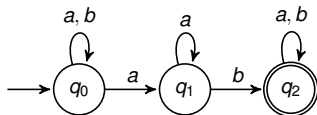
τ_a

$q_0 \quad q_0$

$q_1 \quad q_1$

$q_2 \quad q_2$

Transition monoid



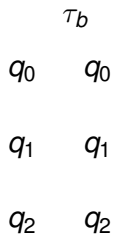
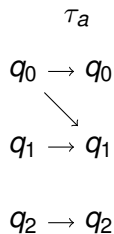
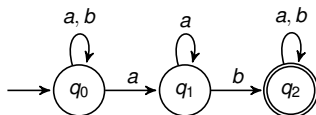
τ_a

$q_0 \rightarrow q_0$

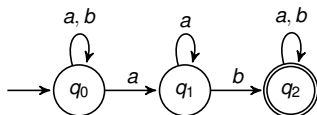
$q_1 \rightarrow q_1$

$q_2 \rightarrow q_2$

Transition monoid



Transition monoid



τ_a

$q_0 \rightarrow q_0$



$q_1 \rightarrow q_1$

$q_2 \rightarrow q_2$

τ_b

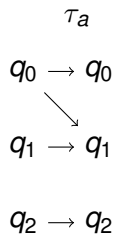
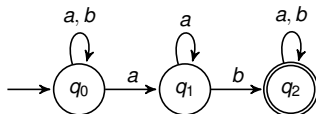
$q_0 \rightarrow q_0$

$q_1 \rightarrow q_1$

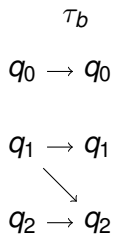


$q_2 \rightarrow q_2$

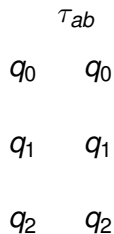
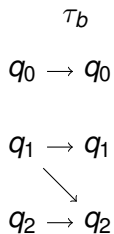
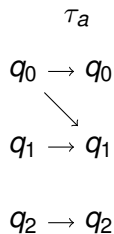
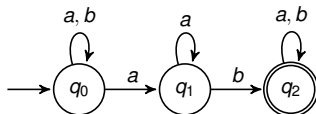
Transition monoid



○

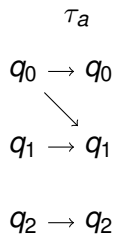
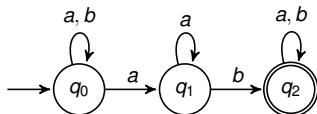


Transition monoid

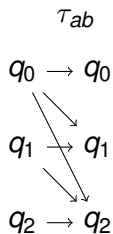
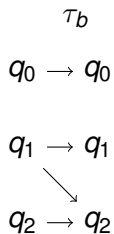


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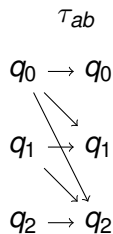
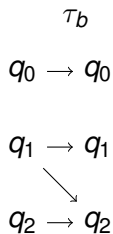
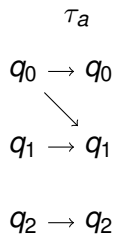
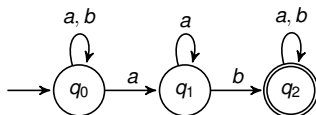
Transition monoid



o



Transition monoid

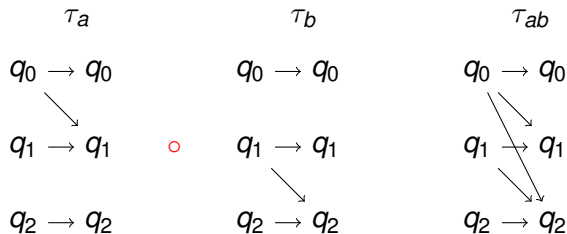
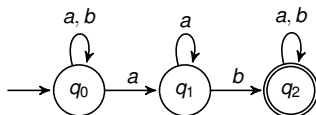


○

Transition inhabitation problem

Given a binary relation τ' over Q , is there any $w \in \Sigma^*$ such that $\tau' = \tau_w$?

Transition monoid



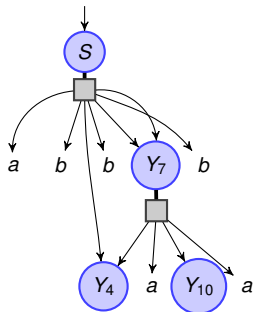
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Inhabitation problem is PSPACE-complete [Kozen 1977].

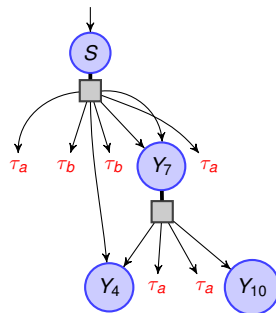
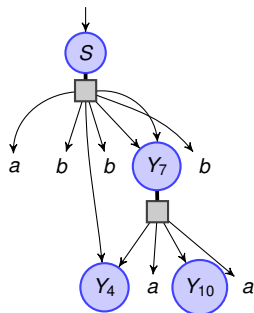
Decision procedure for regular compressed matching problem

Compressed string pattern G and NFA $A = (Q, \Sigma, \delta, I, F)$.



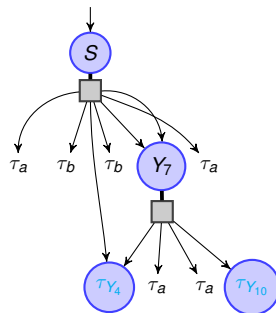
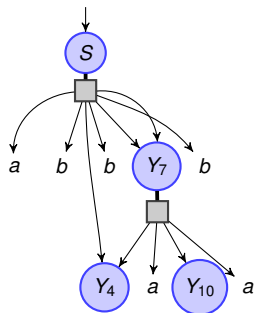
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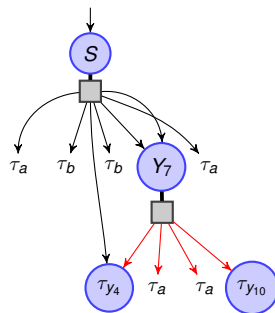
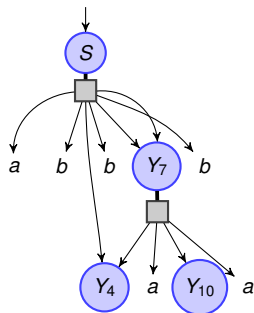
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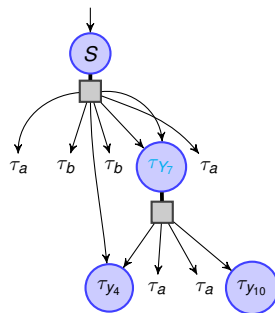
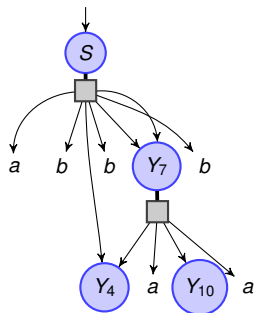
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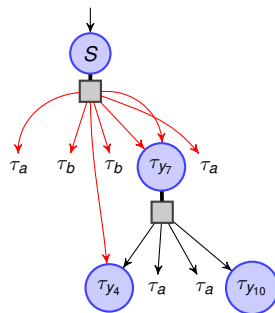
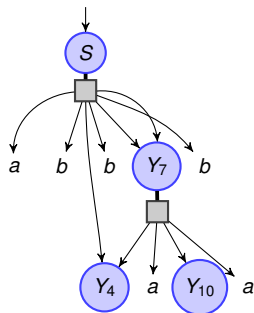
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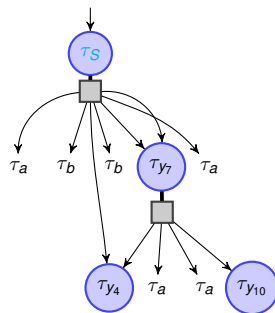
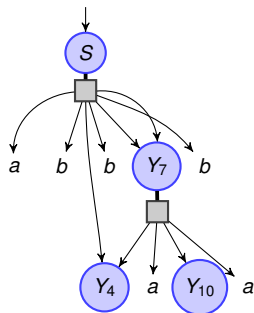
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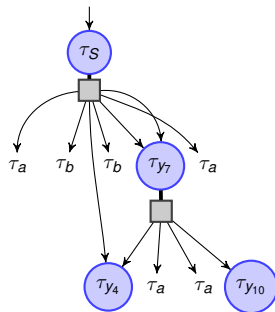
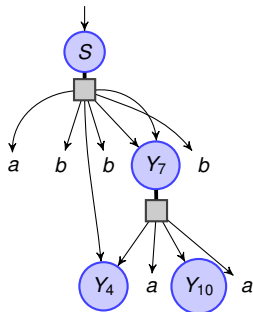
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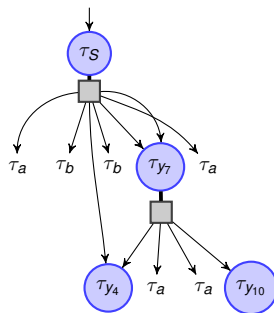
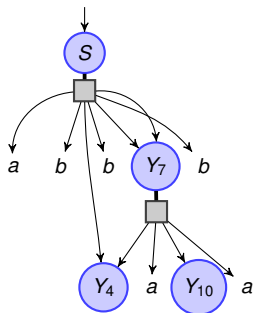
Compressed string pattern G and NFA $A = (Q, \Sigma, \delta, I, F)$.



$$\tau_S \cap (I \times F) \neq \emptyset ?$$

Decision procedure for regular compressed matching problem

Compressed string pattern G and NFA $A = (Q, \Sigma, \delta, I, F)$.



$\tau_S \cap (I \times F) \neq \emptyset ?$
Similar algorithm for Inclusion

Contribution (Complexity of CQA on **Hyperstreams**)

	DFAs	NFAs
Answers (<i>REGINCL</i>)	-	PSPACE-hard
Non-answers (<i>coREGMATCH</i>)	-	-

Contribution (Complexity of CQA on **linear hyperstreams**)

	DFAs	NFAs
Answers (<i>REGINCL</i>)	-	PSPACE-hard
Non-answers (<i>coREGMATCH</i>)	-	-

Contribution (Complexity of CQA on **Hyperstreams**)

	DFAs	NFAs
Answers (<i>REGINCL</i>)	-	PSPACE-C
Non-answers (<i>coREGMATCH</i>)	PSPACE-C	PSPACE-C

Contribution (Complexity of CQA on **linear hyperstreams**)

	DFAs	NFAs
Answers (<i>REGINCL</i>)	-	PSPACE-C
Non-answers (<i>coREGMATCH</i>)	-	-

Proposition

$$\text{REGINCL}(_, \text{DFAs}) = \text{coREGMATCH}(_, \text{DFAs})$$

Proposition

$REGINCL(_, DFAs) = coREGMATCH(_, DFAs)$

Proof idea

For hyperstream G and NFA A , $Inst(G) \subseteq L(A)$ iff $Inst(G) \cap L(\bar{A}) = \emptyset$.
PTIME complementation for DFAs.

Contribution (Complexity of CQA on **Hyperstreams**)

	DFAs	NFAs
Answers (<i>REGINCL</i>)	-	PSPACE-C
Non-answers (<i>coREGMATCH</i>)	PSPACE-C	PSPACE-C

Contribution (Complexity of CQA on **linear hyperstreams**)

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Contribution (Complexity of CQA on **Hyperstreams**)

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Contribution (Complexity of CQA on **linear hyperstreams**)

	DFAs	NFAs
Answers (<i>REGINCL</i>)	-	PSPACE-C
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An efficient fragment

Theorem

REGMATCH(Linear_String_Patterns, NFAs) is in PTIME.

An efficient fragment

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Evaluation with the transition monoid, using the accessibility relation of the automaton as transition for variables.

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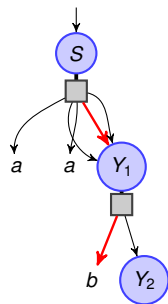
Corollary

REGINCL(Linear_String_Patterns, DFAs) is in PTIME.

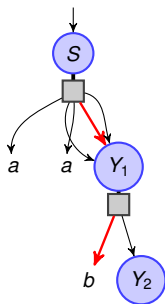
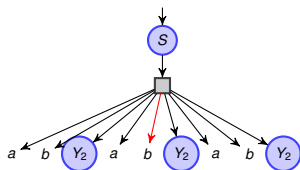
Theorem

CQA(Linear_Hyperstreams, DFAs) is in PTIME.

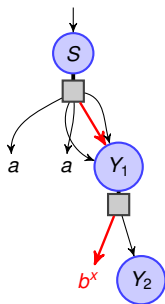
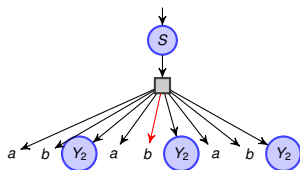
Example of partial decompression



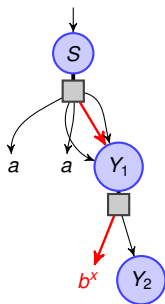
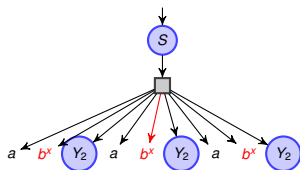
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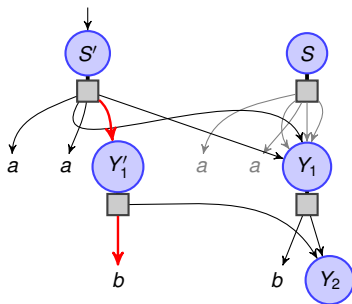
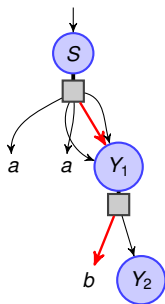
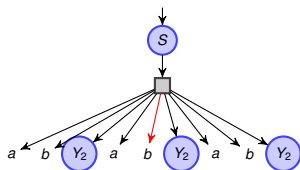
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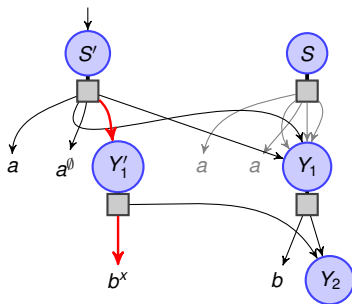
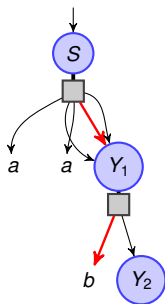
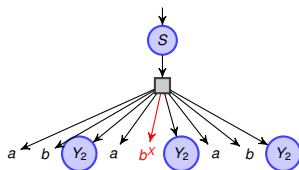
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Example of partial decompression



Contribution (Complexity of CQA on **Hyperstreams**)

	DFAs	NFAs
Answers (<i>REGINCL</i>)	PSPACE-C	PSPACE-C
Non-answers (<i>coREGMATCH</i>)	PSPACE-C	PSPACE-C

Contribution (Complexity of CQA on **linear hyperstreams**)

	DFAs	NFAs
Answers (<i>REGINCL</i>)	-	PSPACE-C
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Contribution (Complexity of CQA on **linear hyperstreams**)

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Answers (<i>REGINCL</i>)	P TIME	PSPACE-C
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- 1 Certain Query Answering on hyperstreams of words
- 2 Decidability and Complexity of Certain Query Answering on Compressed String Patterns
- 3 Future Work**

- Short and middle term
 - Tractable approximations of Certain query answering on NFAs
 - Practical hyperstreaming algorithms
 - Extension to Trees

- Short and middle term
 - Tractable approximations of Certain query answering on NFAs
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 - Extension to transformations
 - Extension to Graphs