News on Safety Properties for Timed Petri Nets

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Universal Safety for Timed Petri Nets is PSPACE-complete

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Abstract

A timed network consists of an arbitrary number of initially identical 1-clock timed automata, interacting via hand-shake communication. In this setting there is no unique central controller, since all automata are initially identical. We consider the universal safety problem for such controller-less timed networks, i.e., verifying that a bad event (enabling some given transition) is impossible regardless of the size of the network.

This universal safety problem is dual to the existential coverability problem for timed-arc Petri nets, i.e., does there exist a number m of tokens, such that starting with m tokens in a given place, and none in the other places, some given transition is eventually enabled.

We show that these problems are PSPACE-complete.

2012 ACM Subject Classification Theory of computation → Timed and hybrid models

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1 Introduction

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A: PSPACE-complete [for $d = 1$, This paper]

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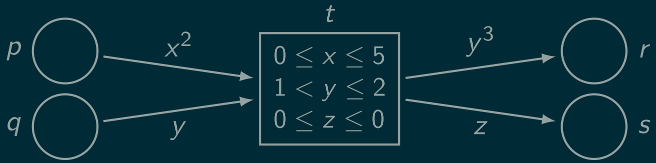

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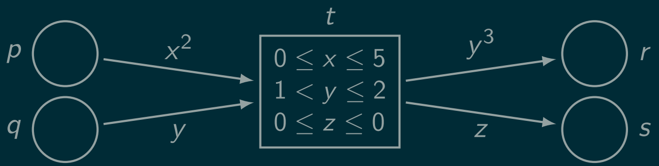

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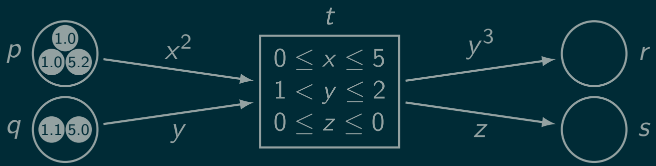

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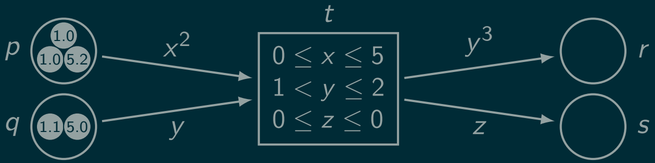

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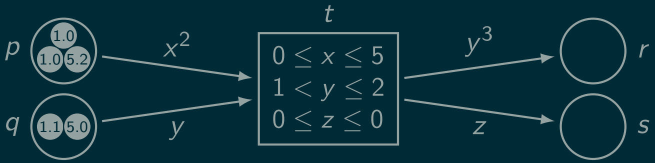

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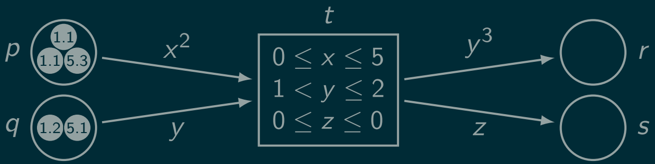

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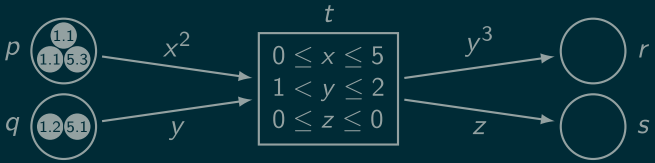

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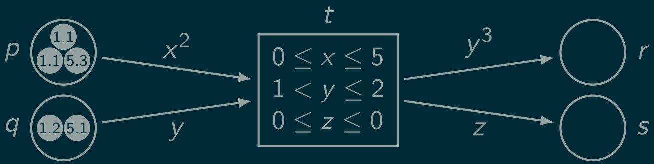

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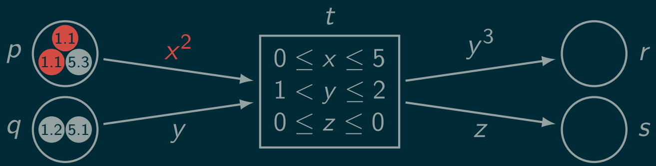

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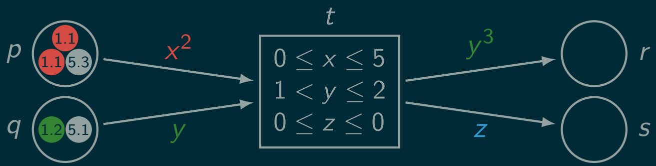

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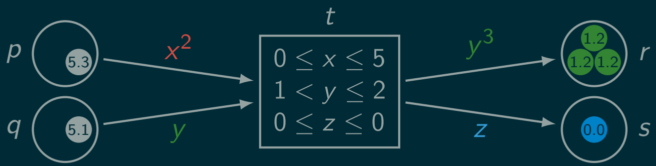

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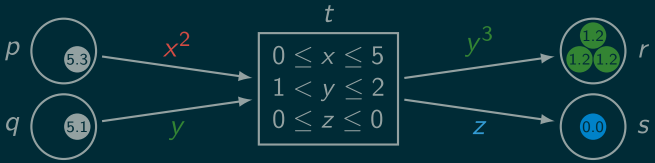

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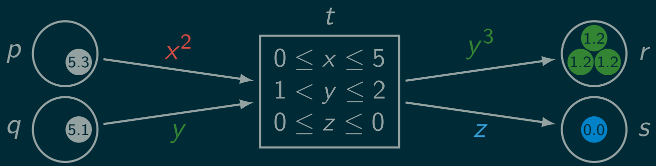

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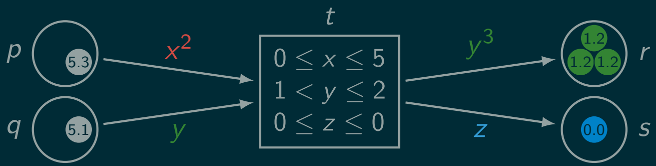

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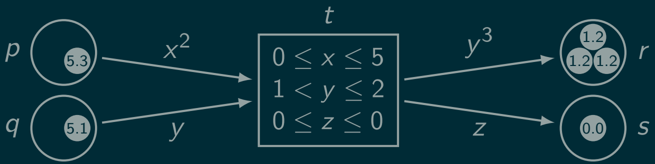

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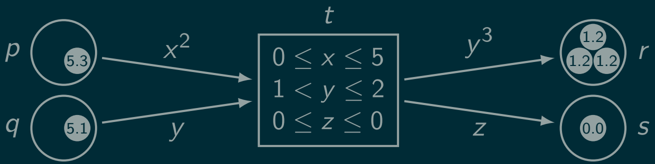

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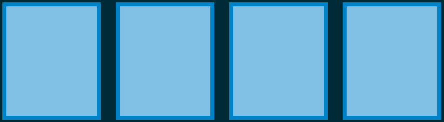

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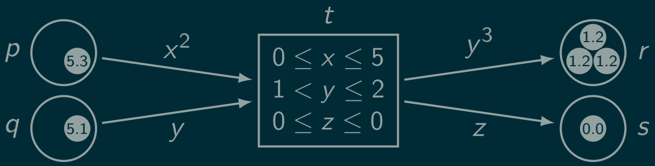

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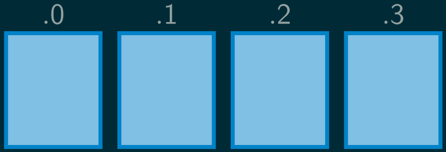

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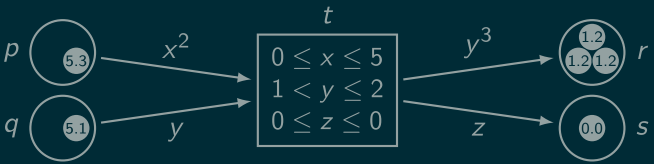

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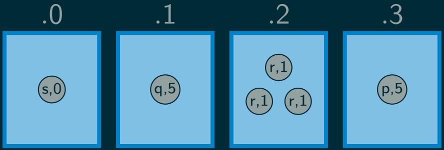

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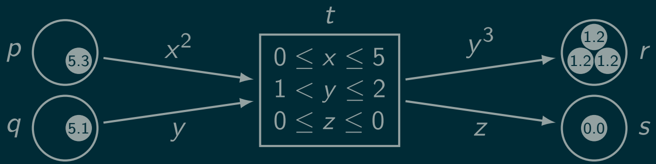

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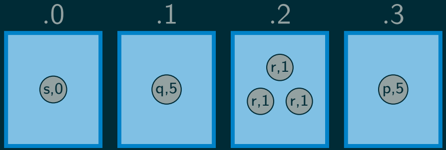

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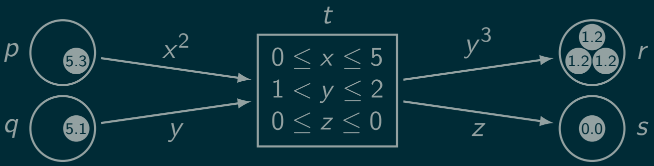

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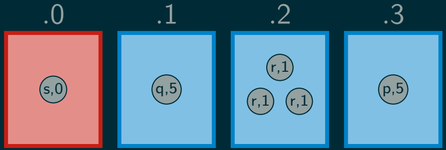

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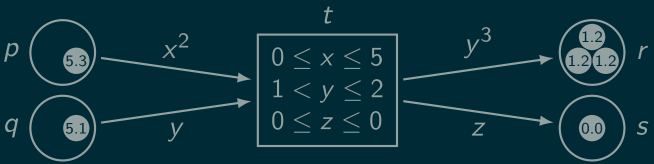

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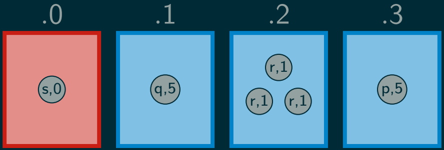

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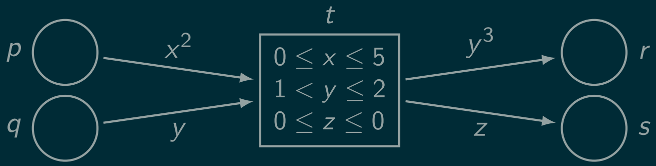

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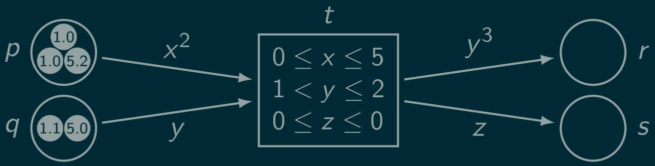

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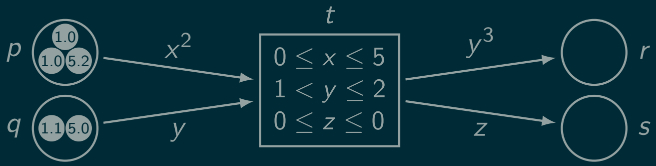

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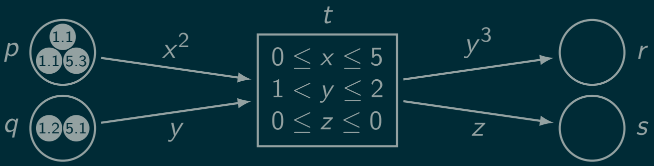

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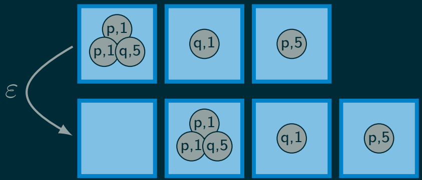

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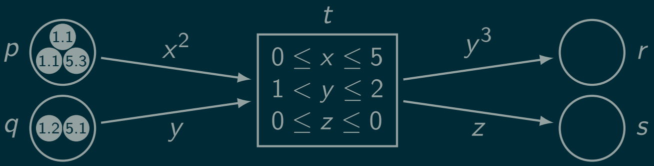

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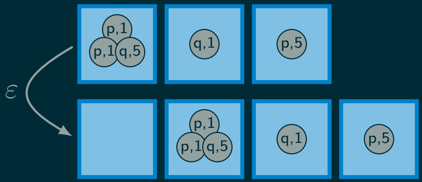

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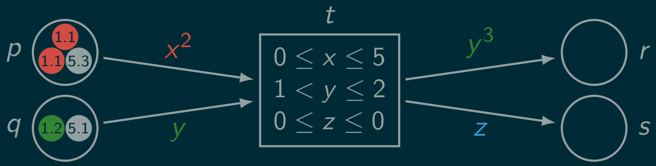

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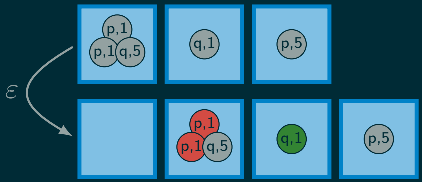

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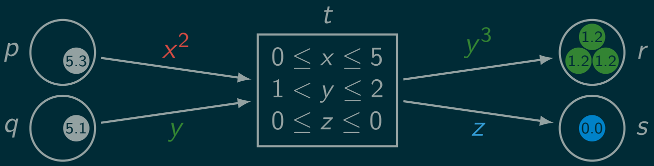

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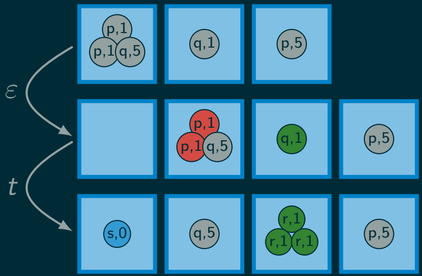

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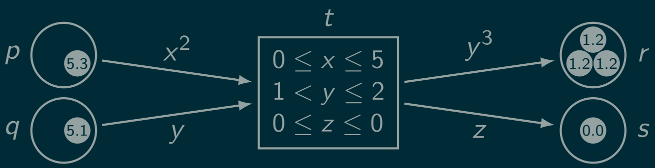

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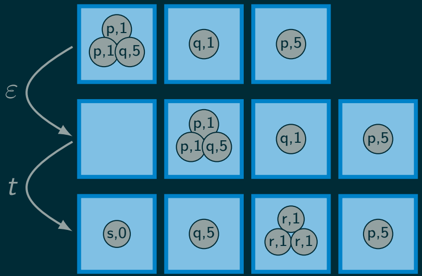

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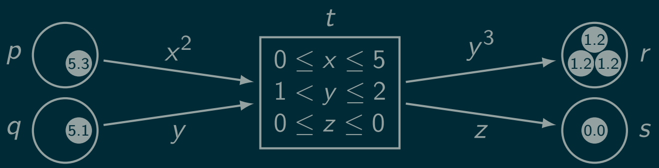

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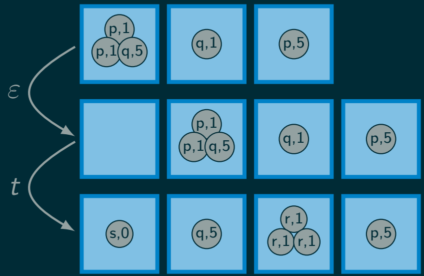

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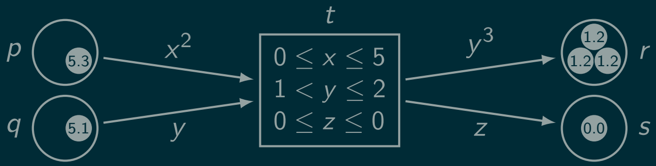

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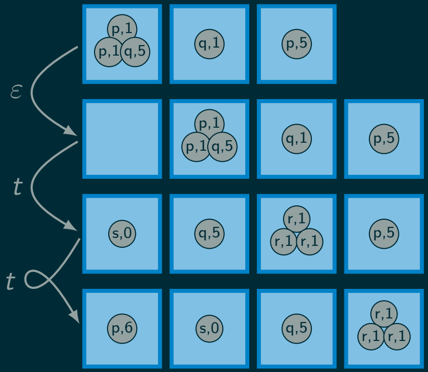

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NB: this fails for $d \geq 2$, for several reasons... Indeed we have undecidability in general.

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≈ Coverability for TPN with continuous firing semantics á la Haddad et al.'17.

– (logspace) reduces to Coverability

– We show PSPACE-completeness

LB: iterated monotone circuits

Existential Coverability

In: A TPN, a marking M, a transition t

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  UB: Regions + forward acceleration

Existential Coverability

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2. Wlog., the net is *non-consuming*: ${}^\bullet t \subseteq t^\bullet$ for all transitions t. This means that discrete transition firing is non-decreasing and for every region R

   - there is a unique maximal region R' with $R \xrightarrow{disc}{}^* R'$
   - R' is (Ptime) computable

Existential Coverability: Key Observation

When forward exploring zeno behaviour regions “stabilize”

A

Existential Coverability: Key Observation

When forward exploring zeno behaviour regions "stabilize"

Existential Coverability: Key Observation

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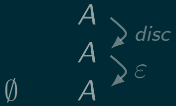

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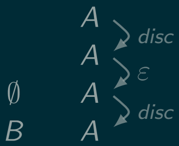

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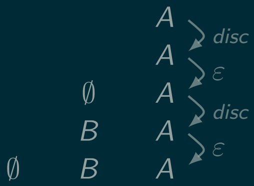

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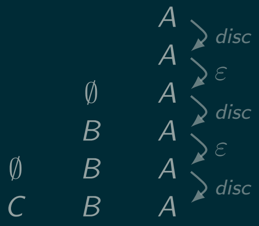

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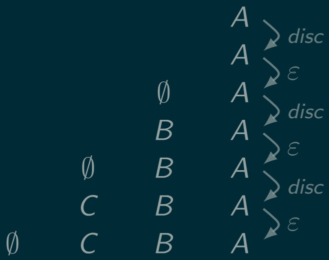

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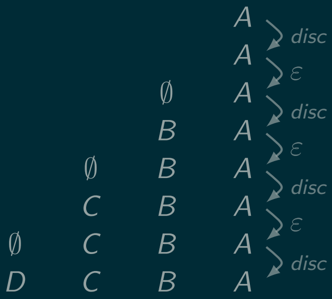

Existential Coverability: Key Observation

When forward exploring zeno behaviour regions "stabilize"

				A	$\downarrow$ *disc*
				A	$\downarrow$ ε
			$\emptyset$	A	$\downarrow$ *disc*
			B	A	$\downarrow$ ε
		$\emptyset$	B	A	$\downarrow$ *disc*
		C	B	A	$\downarrow$ ε
	$\emptyset$	C	B	A	$\downarrow$ *disc*
	D	C	B	A	$\downarrow$ ε
$\emptyset$	D	C	B	A	

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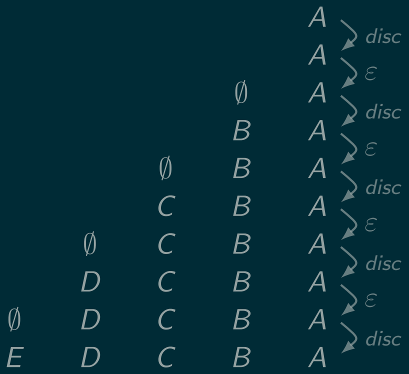

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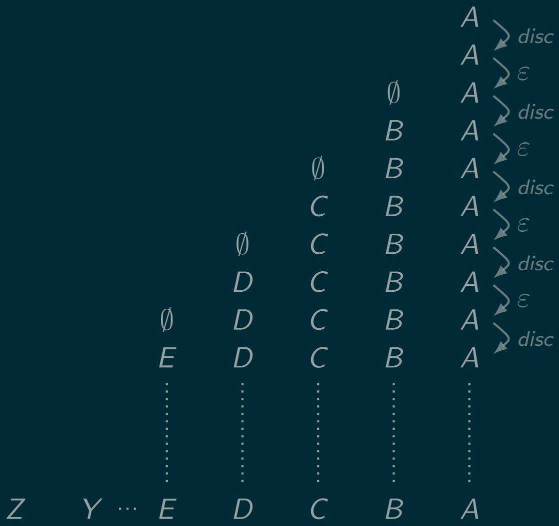

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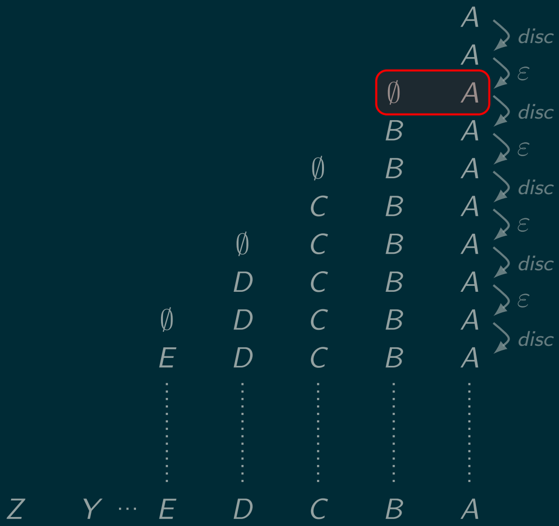

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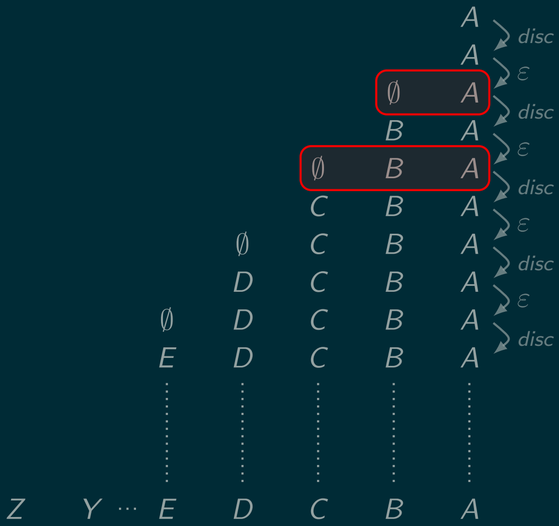

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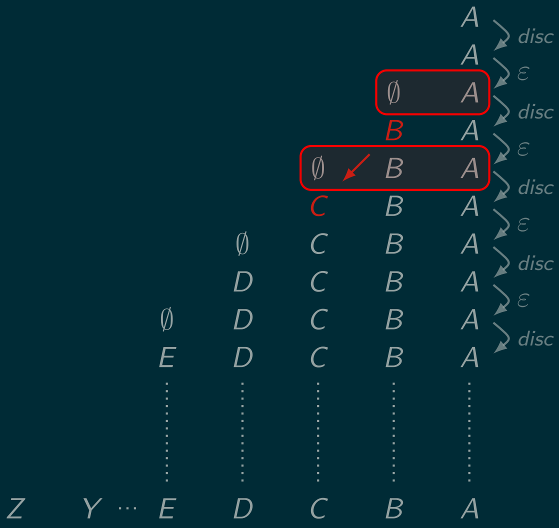

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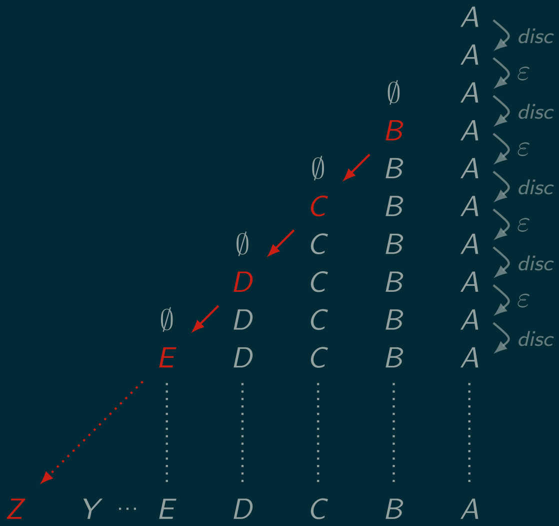

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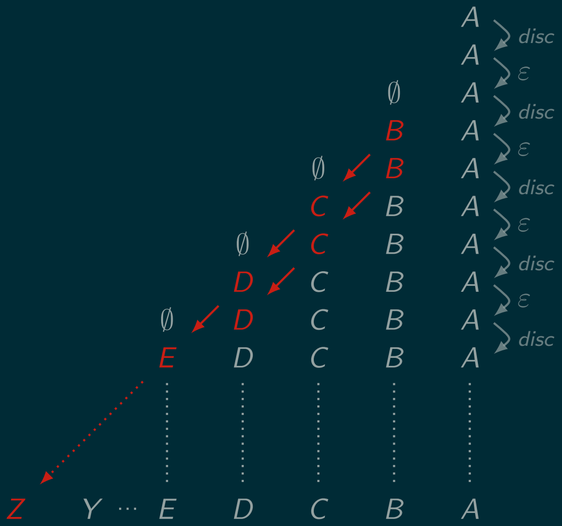

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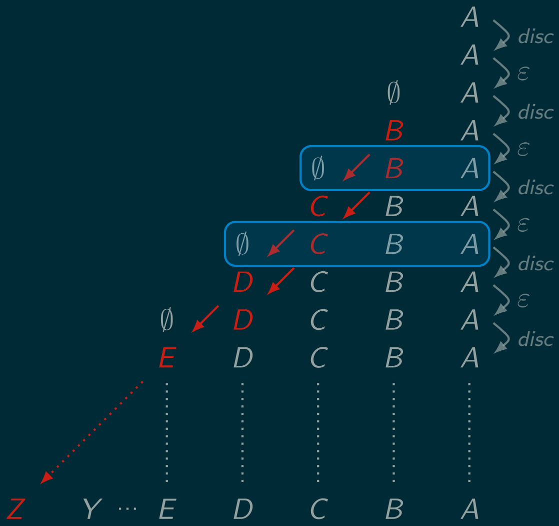

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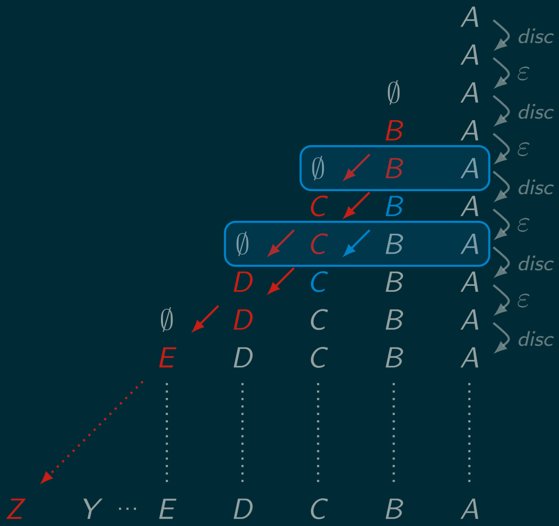

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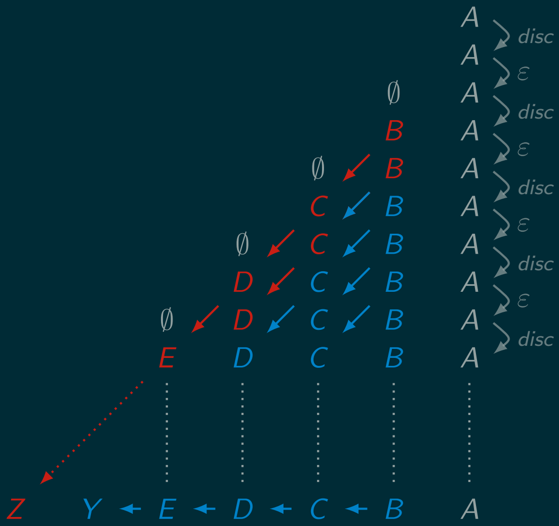

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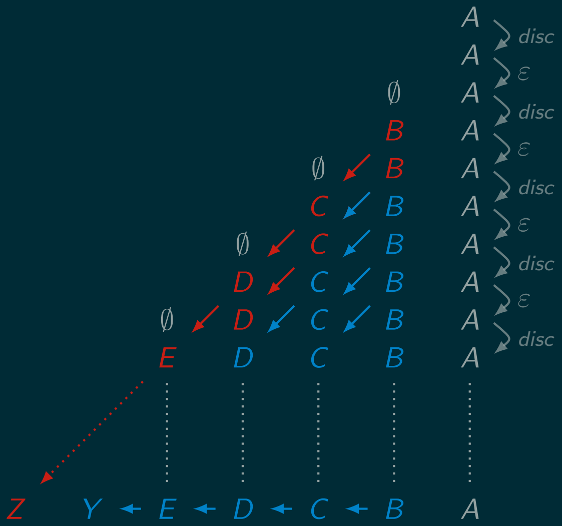

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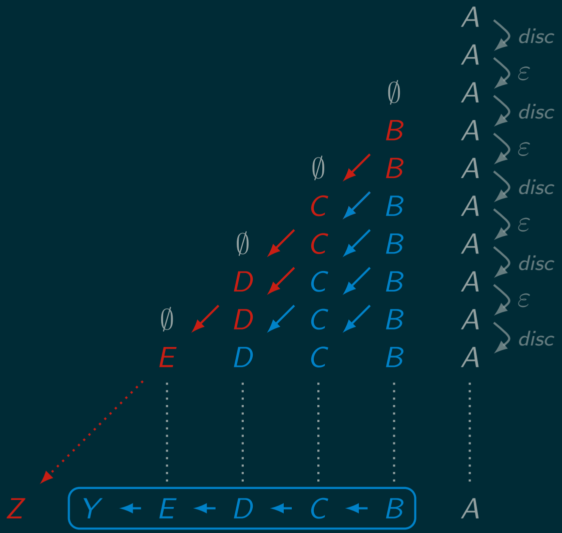

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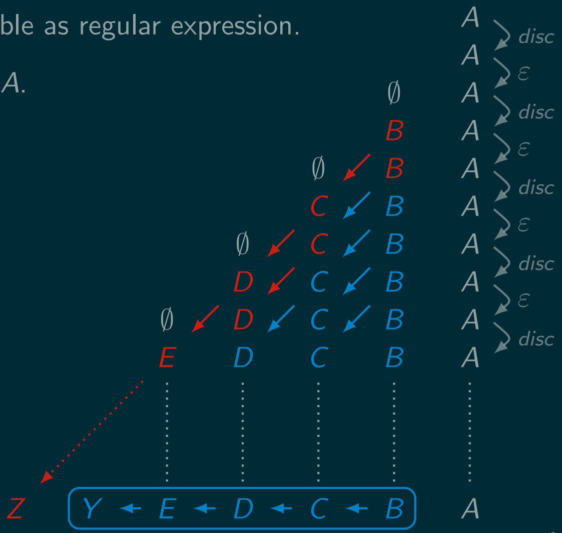
and the limit is expressible as regular expression.

In this example as ZY^*A.



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Forward Exploration

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x_1 x_0^* start

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x_1^1	$(x_0^1)^*$	saturate

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$(x_0^1 + 1)$	x_1^1	$(x_0^1)^*$	rotate

Forward Exploration

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x_2^2	x_1^2	$(x_0^2)^*$	saturate

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	x_2^2	x_1^2	$(x_0^2)^*$	saturate
$(x_0^2 + 1)$	x_2^2	x_1^2	$(x_0^2)^*$	rotate
x_3^3	x_2^3	x_1^3	$(x_0^3)^*$	saturate

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		(x_0^1+1)	x_1^1	$(x_0^1)^*$	rotate
		x_2^2	x_1^2	$(x_0^2)^*$	saturate
	(x_0^2+1)	x_2^2	x_1^2	$(x_0^2)^*$	rotate
	x_3^3	x_2^3	x_1^3	$(x_0^3)^*$	saturate
(x_0^3+1)	x_3^3	x_2^3	x_1^3	$(x_0^3)^*$	rotate

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		x_2^2	x_1^2	$(x_0^2)^*$	saturate
	$(x_0^2 + 1)$	x_2^2	x_1^2	$(x_0^2)^*$	rotate
	x_3^3	x_2^3	x_1^3	$(x_0^3)^*$	saturate
$(x_0^3 + 1)$	x_3^3	x_2^3	x_1^3	$(x_0^3)^*$	rotate
x_4^4	x_3^4	x_2^4	x_1^4	$(x_0^4)^*$	saturate

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		$(x_0^2 + 1)$	x_2^2	x_1^2	$(x_0^2)^*$	rotate
		x_3^3	x_2^3	x_1^3	$(x_0^3)^*$	saturate
	$(x_0^3 + 1)$	x_3^3	x_2^3	x_1^3	$(x_0^3)^*$	rotate
	x_4^4	x_3^4	x_2^4	x_1^4	$(x_0^4)^*$	saturate

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	x_3^3	x_2^3	x_1^3	$(x_0^3)^*$	saturate
$(x_0^3 + 1)$	x_3^3	x_2^3	x_1^3	$(x_0^3)^*$	rotate
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	x_3^3	x_2^3	x_1^3	$(x_0^3)^*$	saturate
$(x_0^3 + 1)$	x_3^3	x_2^3	x_1^3	$(x_0^3)^*$	rotate
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	x_3^3	x_2^3	x_1^3	$(x_0^3)^*$	saturate
$(x_0^3 + 1)$	x_3^3	x_2^3	x_1^3	$(x_0^3)^*$	rotate
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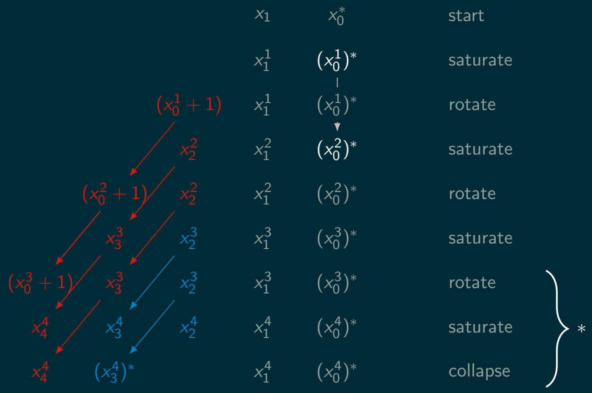
$(x_0^3 + 1)$	x_3^3	x_2^3	x_1^3	$(x_0^3)^*$	rotate
x_4^4	x_3^4	x_2^4	x_1^4	$(x_0^4)^*$	saturate
x_4^4	$(x_3^4)^*$		x_1^4	$(x_0^4)^*$	collapse

Forward Exploration



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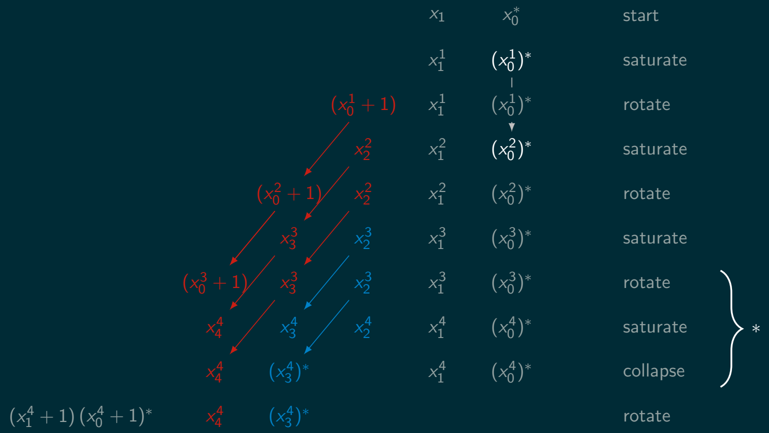

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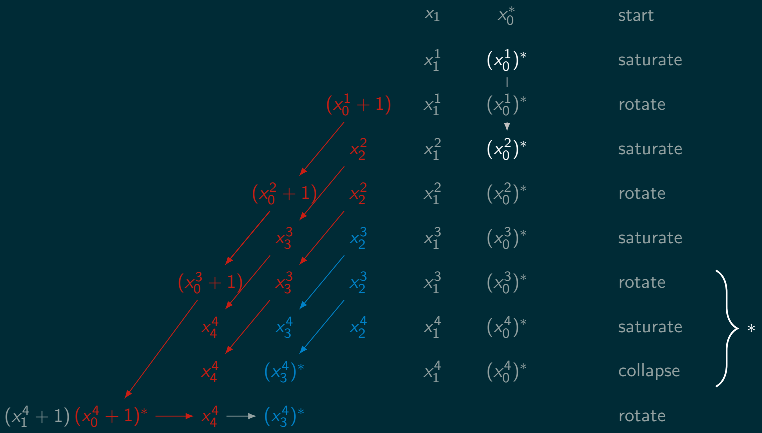

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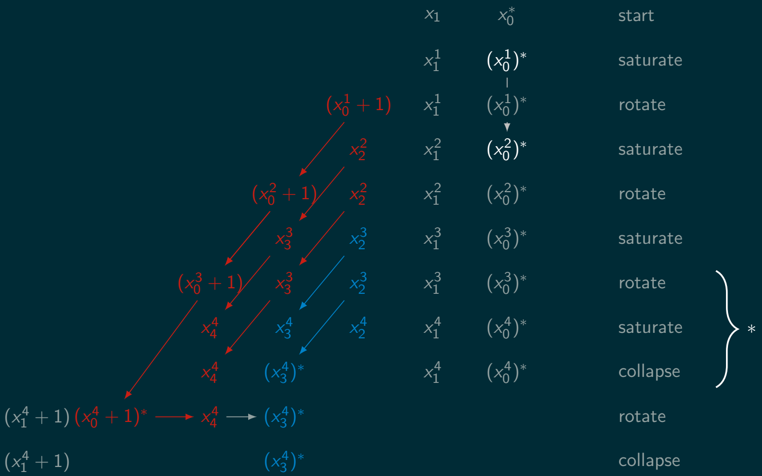

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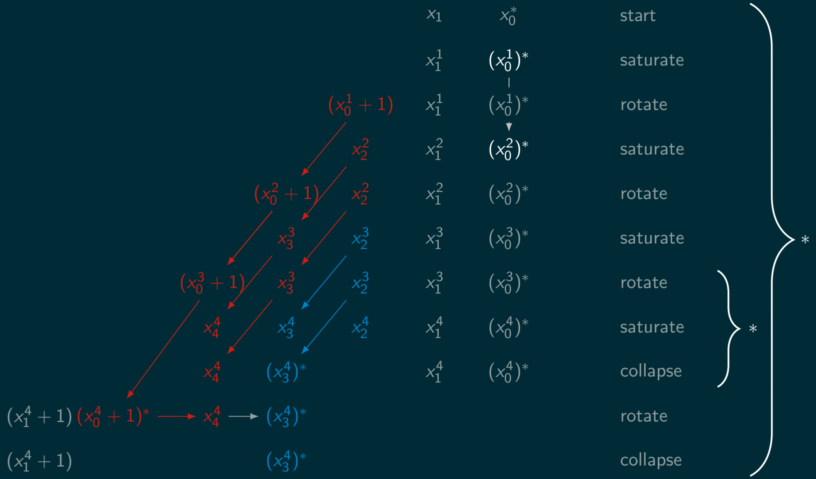

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Corollary

- the sequence is singly exponential
- checking Existential Coverability is in PSPACE.

WIP 1: multi-dimensional ECOVER

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Conjecture

ECOVER is PSPACE-completeness for any fixed dimension d.

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WIP 1: multi-dimensional ECOVER

Conjecture

ECOVER is PSPACE-completeness for any fixed dimension d.

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- Semantics of Kleene stars?

WIP 2: Coverability for TBPP

BPP nets: every transition consumes at most one token

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  - in PSPACE for all d
    (lossy semantics $\rightsquigarrow$ witnesses visit only small regions)
  - conjecture: NP-complete for $d = 1$
- Reachability:
  - 1st step: bound ”time to kill” a region?

thank you.