EXPSPACE-Complete Variant of Countdown Games, and Simulation on Succinct One-Counter Nets

Petr Jančar[1], Petr Osička[1], Zdeněk Sawa[2]

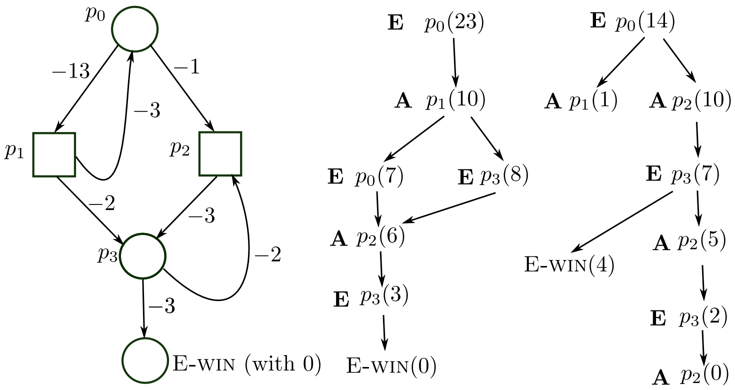
[1]Palacký University Olomouc, Czechia
[2]Techn. Univ. Ostrava, Czechia

12th Int. Conf. on Reachability Problems, Marseille,
25 Sept 2018

- Hofman, P., Lasota, S., Mayr, R., Totzke, P.: Simulation problems over one-counter nets. Logical Methods in Comp. Sci. **12**(1) (2016)
  Open: simulation on concise OCN (PSPACE – EXPSPACE)
  Here: EXPSPACE-complete
- Hunter, P.: Reachability in succinct one-counter games. In: RP 2015
  - Göller, S., Haase, C., Ouaknine, J., Worrell, J.: Model checking succinct and parametric one-counter automata. In: ICALP 2010.
  - Göller, S., Lohrey, M.: Branching-time model checking of one-counter processes. In: STACS 2010.

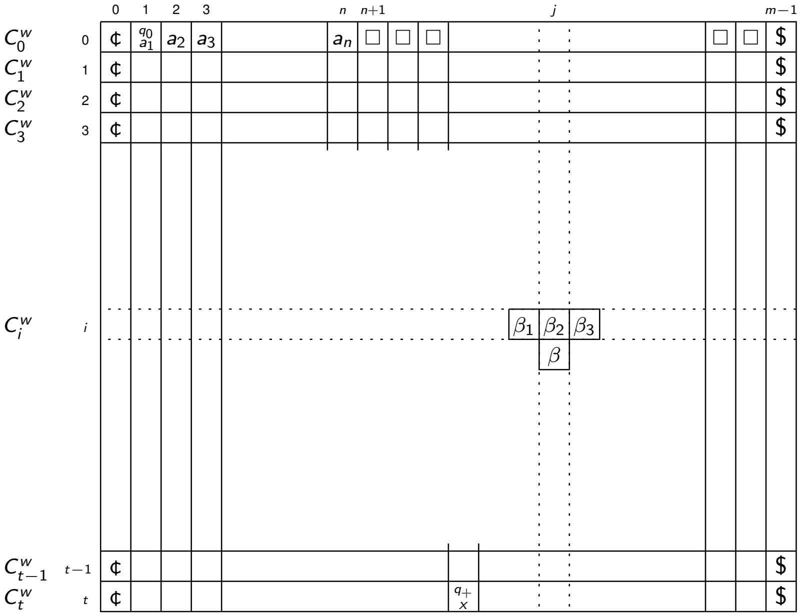
  Here: Reachability game reduces to (bi)simulation relations
- Jurdzinski, M., Sproston, J., Laroussinie, F.: Model checking probabilistic timed automata with one or two clocks. Logical Methods in Comp. Sci. **4**(3) (2008)
  Countdown games ... EXPTIME-complete
  Here: "Existential" countdown games EXPSPACE-complete
  - Chandra, A.K., Kozen, D., Stockmeyer, L.J.: Alternation. J. ACM **28**(1), 114–133 (1981)

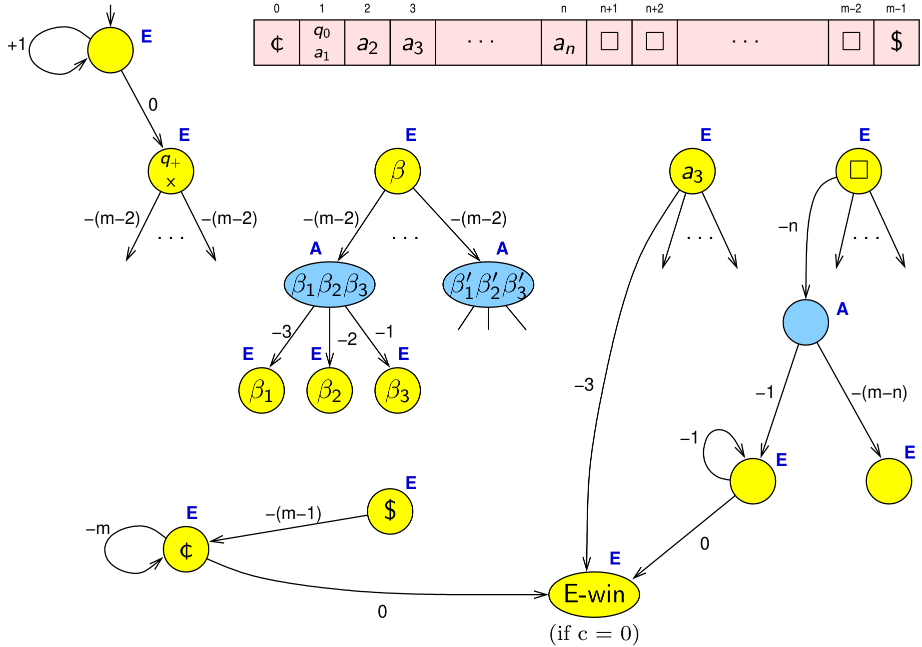
$G = (Q_{\mathrm{E}}, Q_{\mathrm{A}}, \delta)$, δ being a finite set of rules $p \xrightarrow{x} p'$, $x \in \mathbb{Z}$, $x < 0$.



From $p_0(23)$ Eve has a strategy to reach E-WIN(0), from $p_0(14)$ no.

Can Eve force E-WIN(0) from $p(k)$? ... EXPTIME-complete.
Can Eve force E-WIN(0) from $p(k)$ for some k ? ... EXPSPACE-complete.


0 1 2 3 n n+1 j m−1
C_0^w 0 ¢ q_0 a_1 a_2 a_3 a_n □ □ □ □ □ \$
C_1^w 1 ¢ \$
C_2^w 2 ¢ \$
C_3^w 3 ¢ \$
C_i^w i β_1 β_2 β_3
β
C_{t-1}^w t−1 ¢ \$
C_t^w t ¢ q_+ x \$


0
1
2
3
n
n+1
n+2
m−2
m−1
¢
q_0 a_1
a_2
a_3
. . .
a_n
$
+1
E
0
E
q_+ x
−(m−2)
−(m−2)
E
β
−(m−2)
−(m−2)
A
A
$\beta_1\beta_2\beta_3$
$\beta'_1\beta'_2\beta'_3$
−3
−2
−1
E
E
E
β_1
β_2
β_3
E
a_3
E
−n
A
−3
−1
−(m−n)
−1
E
E
E
$
−(m−1)
E
−m
¢
0
0
E
E-win
(if c = 0)

We assume a labelled OCN

$$N = (Q, A, \delta),$$

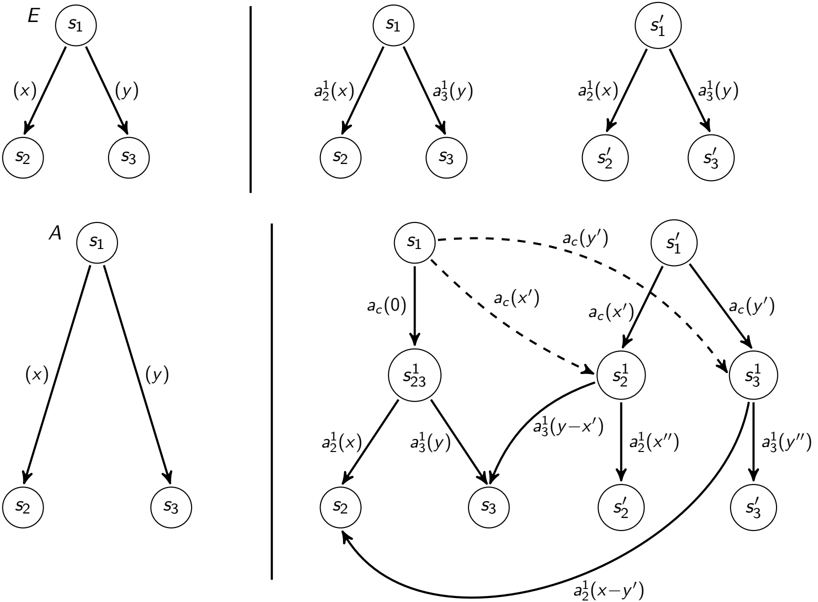
δ being a finite set of rules $p \xrightarrow{a,x} p'$, $a \in A$, $x \in \mathbb{Z}$.
Configurations $p(m) \in Q \times \mathbb{N}$.
Transitions: if $p \xrightarrow{a,x} p'$ is a rule then $p(m) \xrightarrow{a,x} p'(m+x)$ if $m+x \geq 0$.

Simulation game,
In a pair $\big(p(m), q(n)\big)$

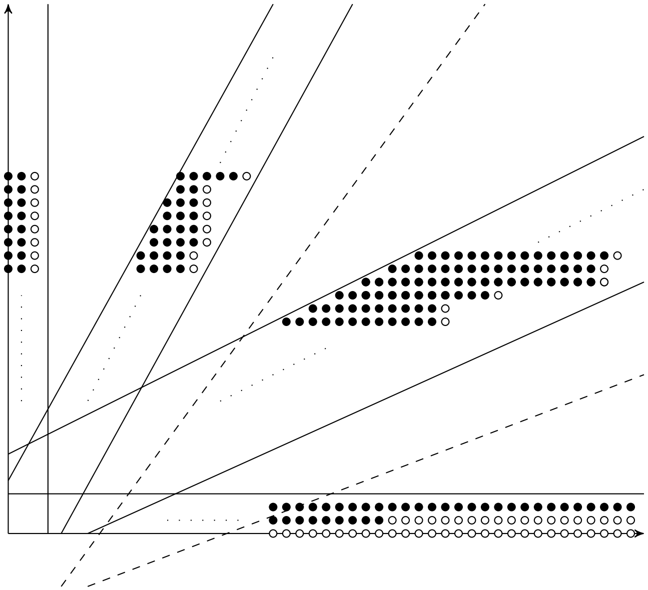
1. Attacker makes a move $p(m) \xrightarrow{a,x} p'(m+x)$
2. Defender responds by $q(n) \xrightarrow{a,y} q'(n+y)$

The play continues from $\big(p'(m+x), q'(n+y)\big)$.
If a player has no possible move, (s)he loses.
An infinite play is Defender's win.

$p(m) \preceq q(n)$ iff Defender has a winning strategy (from $\big(p(m), q(n)\big)$)



$x' = \min\{x, 0\}$, $x'' = \max\{x, 0\}$, and $y' = \min\{y, 0\}$, $y'' = \max\{y, 0\}$



$$P_{<p,q>}(m,n) = \text{BLACK} \text{ iff } p(m) \preceq q(n)$$

Exponential thickness of belts, double-exponential period